Research Summary:
Cognition, Including Math and Science (COG) Domain in the DRDP (2015) Assessment Instrument

The **Cognition, Including Math and Science (COG)** domain focuses on foundational concepts about the identity and relations between objects. The COG domain includes the following knowledge or skill areas: spatial relationships, classification, number sense of quantity, number sense of math operations, measurement, patterning, shapes, cause and effect, inquiry through observation and investigation, documentation and communication of inquiry, and knowledge of the natural world.

COG 1: Spatial Relationships

The measure *Spatial Relationships* describes children's increasing understanding of how objects move in space or fit in different spaces. The measure involves navigating the environment to reach goal locations and attending to objects, people, and own body moving through space (NRC, 2009). Evidence shows that a sense of spatial relationships emerges early in life. Young infants are able to code spatial information about objects, shapes, distances, locations, and spatial relations. For example, three-month-old infants are sensitive to relationships of *above versus below* and *left versus right* (Quinn, 2004). Somewhat later, at six to seven months, they are sensitive to *between* relationships (Quinn, Norris, Pasko, Schmader, & Mash, 1999). Infants and toddlers can also locate objects in space using both landmarks and geometric cues. By 12 months, they typically can code distance and direction and use this information as they move in space to search for hidden objects (Newcombe, Huttenlocher, & Learmonth, 1999). Moving around involves *spatial orientation*, knowing where one is, and how to get around in the environment. As young children move in space, they encode information of their own position, their movements through space, and external references. By the age of two, children can use knowledge of multiple landmarks and distances between them to determine or remember locations. Young children also begin to acquire language for spatial relationships, including terms such as *in, on*, *under*, *up,* and *down.* Children then learn to use words of proximity, such as "beside" and "between" and later terms such as "in front' and "behind" (Greenes, 1999; NRC, 2009)

COG 2: Classification

Classification is the systematic arrangement of objects into groups according to a certain criterion (California Department of Education, 2010a). This measure highlights how children show an increasing ability to compare, match, and sort objects into groups according to their attributes. Early on, infants and toddlers seek to touch and handle objects. They explore toys with hands and mouth, hold, roll, and bang objects. Through observations and by acting on objects, they learn about the attributes of objects (size, shape, material, or weight). At a very young age, they also have different expectations with respect to animate and inanimate objects (Molina, Van de Walle, Condry, & Spelke, 2004). In particular, they expect people to act differently than objects do (Molina et al., 2004) and distinguish between familiar and unfamiliar objects or people (Simion & Giorgio, 2015). They also begin to make a connection between two objects based on similarity or relationships. Research distinguishes between two types of categorization made by young children: perceptual and conceptual (Mandler, 2000). Perceptual categorization involves similarities or differences that can be observed based on physical characteristics, such as similarities in visual attributes like size, color, or shape. Conceptual categorization refers to grouping based on what objects do or how they act.

Young children develop the ability to sort consistently based on a single attribute (e.g., size, color, or shape). They also learn to re-classify by different attributes, understanding that the same collection can be sorted in different ways and that objects can belong to more than one group. The research literature on the

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development of classification emphasizes children's initial differentiation of broad categories of prototypical and concrete objects, and later the extension to atypical exemplars and distinctions based upon functional attributes. Children also learn to use their developing sorting skills to gather information. They may sort and count the number of objects in each group; for example, they may separate red and yellow apples in two bowls and report which group has more apples.

COG 3: Number Sense of Quantity

The measure *Number Sense of Quantity* represents the child's developing understanding of number and quantity. Research in cognitive development indicates that children are sensitive to quantity beginning in early infancy. Infants as young as five months of age are sensitive to number and are able to discriminate among small sets of up to three objects (Starkey, Spelke, & Gelman, 1990). For example, they can discriminate one object from two, and two objects from three. The ability to quickly and accurately recognize the quantity in a small set of objects without counting is called subitizing. Infants also attend to quantity relations (same, more than, less than) and respond to changes in the number of objects observed (Feigenson, Dehaene, & Spelke, 2004).

As children's understanding and use of language increases, they begin to use numerical language to communicate their non-verbal knowledge of number and quantity. Between 18 and 24 months of age, children use relational words to indicate "more" or "same" as well as number words. They begin to count aloud, typically starting with "one" and continuing with a stream of number names (Fuson, 1988; Gelman & Gallistel, 1978), although they may omit some numbers and not use the conventional number list (e.g., "one, two, three, seven, nine, ten"). The three major basic building blocks for counting are learning (1) the sequence of number words, (2) one-to-one correspondence, and (3) cardinality (knowing that the last number assigned to the last object counted gives the total number in the set) (Gelman & Gallistel, 1978; Wynn, 1992b). Children begin to count small collections of objects but may point to the same item twice or say a number word without pointing to an object. With repeated counting experiences, children learn to apply counting skills precisely and use counting to determine the number of objects in a set (Fuson, 1988). They begin to construct an understanding that the last number word used when counting represents the total number of objects. The ability to count with understanding allows children to solve simple everyday problems involving numbers by using counting (Zur & Gelman, 2004). By age five, most children can say the number words up to 20 accurately, and can count 20 to 30 objects (Sarama & Clements, 2009).

COG 4: Number Sense of Math Operations

The developmental processes highlighted in this measure involve children's ability to reason about quantity and to add and subtract small quantities of objects. Research indicates that the ability to reason about numbers starts as early as young infancy (Wynn, 1992a). Five-month-olds show sensitivity to the effects of addition or subtraction of items on a small collection of objects. Children as young as two years understand that addition increases the number of objects in a set, subtraction decreases the number of objects in a set, and number-irrelevant operations (e.g., spatial rearranging) do not change the number of objects (Gelman & Gallistel, 1978; Zur & Gelman, 2004; Wynn, 1992a). For example, toddlers viewing three balls put into a container and then one being removed know to search for a smaller number of balls, and many search for exactly two balls (Starkey, 1992).

During the preschool years, children are able to solve simple addition and subtraction problems using a variety of strategies. They may use objects or fingers to represent numbers in the problem and count out loud to find out the answer (Levine, Jordan, & Huttenlocher, 1992). As they grow older, they may invent different solutions to solve such problems (Siegler, 1989; Fuson, 1988). For example, to solve a simple addition problem with concrete objects, they count the two different sets starting from "one" to determine the answer (the counting-all strategy); therefore, the development of number operations is closely related to the way they learn to count. As children gain experiences, they may "count on" from the second set of objects. Knowing the number of objects in the first set (e.g., four), the child starts with "four" and continues to count "five, six" to find out the total number of objects (4 + 2) rather than starting to count from "one." Children also develop a basic understanding of part-whole relationships. They recognize that parts can be combined to make a whole, and a whole quantity can be broken into two or more parts (e.g., combining "two" and "four" to make "six" and decomposing "six" into "four" and "two"). Children's experience with the part-whole relationships and the decomposition of numbers into smaller groups forms the conceptual basis for understanding number relationships and operations.

COG 5: Measurement

This measure highlights the ways in which the child shows an increasing understanding of measurable properties such as size, length, weight, and capacity (volume), and how to quantify those properties. Children initially learn to use words that represent quantities or magnitude of a certain attribute. Then, children begin to demonstrate an ability to compare two objects directly and recognize equality or inequality. For example, they may compare two objects to determine which is longer or heavier. After comparing two items, children develop the ability to compare three or more objects and to order them by size (e.g., from shortest to longest) or by other attributes. At ages four through five, most children make progress in reasoning about and measuring quantities. This development sets the foundation for learning to measure and connect numbers to attributes of objects, such as length, weight, amount, and area (Clements, 2004; Ginsburg, Inoue, & Seo, 1999).

Children's familiarity with the language required to describe measurement relationships—such as longer, taller, shorter, the same length, holds less, holds the same amount—is an important foundation for learning the concept of measurement (Greenes, 1999). Young preschoolers learn to use words that describe measurement relationships as they compare two objects directly to determine equality or inequality, and as they order three or more objects by size. Older preschool children begin to make progress in reasoning about measuring quantities with less dependence on perceptual cues (Clements, 2004; Clements & Stephan, 2004). For example, they can compare the length of two objects by representing them with a third object. They can also begin to measure by repeated use of units (e.g., same-size blocks or footsteps) to find the length of objects, and associate higher counts with longer objects (NRC, 2009).

COG 6: Patterning

Patterning involves the natural tendency to organize information in the environment (California Department of Education, 2010b). From the earliest years, children are sensitive to patterns of actions, behaviors, sounds, and visual displays. The ability to recognize patterns contributes to development across domains, including broader social development. Through an understanding of patterns, children are able to make predictions about what comes next. For example, predicting what comes next after eating lunch (cleaning up) or after taking a bath (putting on clean clothes) will help a child build confidence in his or her ability to navigate the environment. Identifying and applying patterns helps bring order and predictability to seemingly unorganized situations and allow for making generalizations (Clements & Sarama, 2009).

This measure highlights the developmental processes underlying the young child's abilities to recognize, reproduce, and create patterns of varying complexity. An explicit understanding of patterns develops gradually during the early childhood years. To understand a pattern in a regularly repeated arrangement of objects, events, or shapes, children need to be able to identify similarities and differences among elements of a pattern, note the repeatable unit, and make generalizations. The developmental trajectory of patterns has been characterized as evolving from the ability to identify and duplicate repeating patterns to children's ability to extend and create patterns (Klein & Starkey, 2004). The perception of the initial unit plays a fundamental role in both the duplication and extension of patterns. Once children are able to identify the repeated unit of a pattern (e.g., AB in a pattern ABABAB), they can extend a pattern by predicting what comes next. In addition to children's ability to identify patterns in arrangements of objects, their ability to engage in rhythmic patterns (jump, jump, fall) and musical patterns (clap, clap, tap) has been demonstrated and described (Clements & Sarama, 2009). Children can express patterns through different modalities represented through different formats (objects, language, sound, movement). Over time, preschool children can identify and create more complex patterns, such as ones with more than two items in the repeating units (e.g., ABC ABC ABC) or patterns with varying number of each element (e.g., AAB AAB) (Smith, 2006).

COG 7: Shapes

This measure highlights the ways in which a child shows an increasing knowledge of shapes and their characteristics. Shape knowledge involves not only recognition and naming but also an understanding of shape characteristics and properties. Van Hiele's (1986) research highlights that children first identify shapes at the visual level on the basis of their appearance. As the National Research Council (2009) describes, at this visual/holistic phase, children recognize shapes as wholes but do not think about shapes in terms of their attributes or properties. They can name a circle, a square, or a triangle (NRC, 2009). However, children's early shape categories center on prototypes, and the similarity of perceptual surface qualities of a shape are used to determine category inclusion. For example, young preschoolers do not accept an inverted triangle as a triangle or non-isosceles triangles as triangles (e.g., Clements, Swaminathan, Hannibal, & Sarama, 1999). They may regard squares as a distinct category and not as a special kind of rectangle with four sides that are equal in length. Next, children represent shapes at the "descriptive" level on the basis of their properties. They learn to describe and analyze geometric figures and recognize and characterize shapes by their properties. They attend to parts such as the number of sides in a square or a rectangle. At the age of four, they typically begin to recognize shapes in different orientations, sizes, and configurations (e.g., "skinny" rectangles) and as part of other objects.

Clements (2004) recommends that young children be given the opportunity to work with many varied examples of a particular shape and many "non-examples" of a particular shape. For example, children learn through experiencing examples of triangles that are not just isosceles triangles to support the development of a robust and explicit sense of the properties of a triangle.

COG 8: Cause and Effect

The *Cause and Effect* measure addresses the ability to relate events causally and identify cause-and-effect relations. Much of the ability to organize information in the environment and identify patterns of human behavior and the relationships between events and consequences rests on the ability to identify cause-and-effect relations. Even very young infants possess expectations about physical events and try out different behaviors to cause effects (Baillargeon, 2004). Everyday experiences such as shaking a rattle to make a sound provide infants with opportunities to construct knowledge of corresponding events and to become aware of cause-and-effect relationships (Kamii & Devries, 1993; DeVries, Zan, Hildebrandt, Edmiaston, & Sales, 2002). At a very young age, children understand the causal relations involved in everyday physics. They typically assume that physical events have a cause and intuitively search for a cause. They are also sensitive to the temporal ordering of cause and effect and believe that causes must precede their effects. They act on objects to cause a specific result. Furthermore, young children can reason about the kind of mechanism that can or cannot produce a certain outcome (such as pulling, pushing, or rolling). They can anticipate results of certain actions and offer explanations for why certain actions or behaviors result in specific effects. For example, when observing a ball as it rolls against a jack-in-the-box, children could reason about the cause and effect. They attribute the effect to the ball hitting the jack-in-the-box, presumably because rolling and hitting can produce movement in another object through impact. When asked to explain how an event occurred, some children were found to generate mechanistic, physically oriented explanations (Bullock, Gelman, & Baillargeon, 1982; California Department of Education, 2012). Finally, they understand that by varying their degrees of actions, they can cause different results (e.g., "if I kick the ball harder, it will go really far"). Everyday experiences in which variations in the child's action result in corresponding variations in an object's reaction provide opportunities to construct knowledge of corresponding events and to deepen understanding of cause-and-effect relationships (Kamii & DeVries, 1993; DeVries et al., 2002).

COG 9: Inquiry through Observation and Investigation

The measure *Inquiry through Observation and Investigation* focuses on children's ability to observe and investigate objects and events in their everyday environment. Early on, infants and toddlers show interest and actively seek information about objects and events. They engage in exploratory play and touch and examine objects using all of their senses. Through observations and by acting on objects, they learn about the physical attributes of objects (size, shape, material, or weight) and how living and non-living objects interact, move, and change. This information forms the foundation for acquiring knowledge about the natural and physical worlds (California Department of Education, 2012). During the preschool years, with adult guidance, young children's simple, purposeful explorations of familiar objects in the environment can evolve into systematic investigations involving sustained explorations, detailed observations, and complex investigations of objects and events. Preschool children are developmentally ready to engage in systematic investigations and use scientific inquiry skills. They can make detailed observations, ask questions, plan investigations, gather and interpret information, and communicate findings and ideas (Gelman & Brenneman, 2004; French, 2004; Gelman, Brenneman, Macdonald, & Roman, 2010). Their scientific investigations are largely based on systematic observations. To expand their observations, they may use scientific tools (e.g., magnifying glasses). They may recognize similarities and differences between one object and another and make comparisons. As children develop their inquiry skills, they can make predictions based on prior knowledge and observable information and test their predictions through simple experiments (Jones, Lake, & Lin, 2008; California Department of Education, 2012). In domains in which young children have conceptual knowledge, their predictions tend to be relatively reasonable and accurate (Bullock, Gelman, & Baillargeon, 1982; Inagaki & Hatano, 2002; Zur & Gelman, 2004). Young children can also make inferences or draw conclusions based on observable evidence or prior knowledge (Gopnik, Sobel, Schulz, & Glymour, 2001).

COG 10: Documentation and Communication of Inquiry

The measure *Documentation and Communication of Inquiry* pertains to processes and skills employed to document and record observations and to communicate ideas and explanations to others. Preschool children learn to use language to describe their observations and communicate their thoughts. The use of language to communicate simple observations and other steps in the exploration process is an integral part of children's learning and formation of scientific concepts (Gelman & Brenneman, 2004; Eshach, 2006). Language extends and enriches scientific experiences and facilitates conceptual growth. For example, as children explore concepts such as growth, nutrition, or weather, they gradually learn the terms for the concepts they explore (e.g., words such as seeds, petals, or roots). They acquire vocabulary to describe in greater detail what they are observing (nouns such as seeds, petals, or roots) and adjectives to describe characteristics of the objects they observe (e.g., pointy, round, soft). The use of these terms, in turn, enriches their learning experiences. Children may also begin to use relevant scientific terms—for example, "I *observe*," "My *prediction* is," and "Let me *check*"—as they practice inquiry skills across a variety of settings (Gelman & Brenneman, 2004). English learners, for whom the development of new vocabulary and language skills is most effective in authentic learning experiences, especially benefit. In the context of scientific explorations, children also learn to build their communication skills and engage in complex discussions involving observation, prediction, and explanation (Peterson & French, 2008). Such discussions develop children's understanding of the scientific phenomena they explore (Jones, Lake, & Lin, 2008) (California Department of Education, 2012). Communication and documentation of inquiry can take different forms. Preschool children can use drawings, photos, graphs, charts, and journals to record and document information in greater detail. For example, they can use drawings and words to document the growth of their plant over time or the transformation of a caterpillar to a butterfly (California Department of Education, 2012, p. 55).

COG 11: Knowledge of the Natural World

The measure *Knowledge of the Natural World* pertains to children's understanding of the essential properties of natural objects (living and non-loving things) and natural events. At a young age, children learn about core concepts related to properties and characteristics of living things. They actively explore and observe the characteristics of animals and plants in the everyday environment, and can study their appearance (insides and outsides), body parts, behaviors, habitats, and the changes and growth of living things over time (Gelman et al., 2010; Worth & Grollman, 2003). They also observe and become aware of the earth's resources (water, sand, rocks) and phenomena (e.g., changes in the weather), but they are not ready to grasp scientific concepts and explanations of some earth phenomena such as the cause of the day/night cycle rotation or of seasonal changes (Kampeza, 2006).

Research indicates that young children have different expectations with respect to animate and inanimate objects. They expect animate objects, but not inanimate objects, to move on their own (Massey & Gelman, 1988), to express emotions (Gelman, Spelke, & Meck, 1983), and to have blood or bones on the inside (Gelman, 1990; Subrahmanyam, Gelman, & Lafosse, 2002). They also have intuition about the essential properties of living objects (plants and animals) and can distinguish them from non-living objects. For example, they realize that animals and plants can grow and increase in size over time (Rosengren, Gelman, Kalish, & McCormick, 1991; Hickling & Gelman, 1995). As children become older (around the age of five) they recognize that plants, but not objects, are similar to animals in terms of growing, becoming older, and dying (Inagaki & Hatano, 1996; California Department of Education, 2012, p. 90). Preschool children understand some aspects of *growth* in animals and plants. They associate growth of plants and animals with feeding and watering, and associate the growth of plants with natural processes such as sunshine and rain (Gelman, 2003). By three years of age, children realize that growth in animals involves an increase in size over time. They realize that growth in animals is affected by food intake, not by an intention or desire to grow (Inagaki & Hatano, 2002). Older children, approximately five years of age, also expect some animals to change in appearance with age and understand that animals undergo metamorphosis—for example, caterpillars change into butterflies, and tadpoles change into frogs (Rosengren, Gelman, Kalish, & McCormick, 1991). Between the ages of four and five, children develop increasing knowledge about plants, including an understanding of some of the characteristics of plant growth and the nature of seeds (Hickling & Gelman, 1995). Over time, by studying and comparing the needs of different animals and plants, young children develop an understanding that elements like food, water, and air are basic needs to *all* living things.

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